Getting bounds on the mixing angles for a non-sequential bottom quark

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Abstract

We analyze the $Zf\overline{f}$ vertex in the framework of models that add a new bottom quark in a nonsequential way and we evaluate the tree level contribution to the LEP/SLC observables Γ_Z, R_l and R_b. We obtain bounds for the mixing angles from the experimentally allowed contour regions of the parameters $\Lambda_{L,R}$ introduced here. In order to get a more restrictive region, we consider the experimental results for $B \to \nu\overline{\nu}X$ as well.

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1 Introduction

The comparison of theoretical predictions with experimental data has confirmed the validity of the Standard Model (SM) in an impressive way. The quantum effects of the SM have been established at the 1σ level, and the direct and indirect determinations of the top quark mass are compatible with each other. In spite of this success, the conceptual situation with the SM is not completely satisfactory for a number of deficiencies. Some of them are the large number of free parameters and the hierarchical fermion masses.

The SM contains three generations of quarks in irreducible representations of the gauge symmetry group $SU(3)_C\times SU(2)_L\times U(1)_Y$. The possibility of extending them has been studied in different frameworks [1]-[8] which are based either on a fourth generation sequential family, or on non-sequential

fermions, regularly called exotic representations because they are different from those of the SM.

These unusual representations emerge in other theories, like the E_6 model where a singlet bottom type quark appears in the fundamental representation [2]; also, top-like singlets have been suggested in supersymmetric gauge theories[3]. The principal feature of a model which extends the quark sector with an exotic fermion is that there are new quark mixing phases in addition to the single phase of the SM. Therefore, in this kind of models Z boson mediated FCNC's arise at tree level. This fact can affect the mixing mechanism in the neutral B-system [1]-[6].

The possibility of indirect consequences of singlet quark mixing for FCNC and CP violation has been used to get bounds on the flavor changing couplings. Heavy meson decays like B^0 and $D^0 \rightarrow \mu^+\mu^-$ [1], [5], rare decays $b \rightarrow s\gamma$ [1], [2], [5], measurements like $K_L \rightarrow \mu^+\mu^-$, $B \rightarrow \mu^+\mu^- X$, $B \rightarrow \nu\overline{\nu}X$, K meson physics [1],[2], [4], or even $Z \rightarrow l\overline{l}$, $l \rightarrow ll\overline{l}$ [9] have been considered for this purpose.

In the last years, the LEP and SLC colliders have brought to completion a remarkable experimental program by collecting an enormous amount of electroweak precision data on the Z resonance. This activity, together with the theoretical efforts to provide accurate SM predictions have formed the apparatus of electroweak precision tests [10]. We are interested in using the electroweak precision test quantities in order to get bounds on the mixing angles for additional fermions in exotic representations. Specifically, we want to consider models that include a new quark with charge $-1/3$ which is mixed with the SM bottom quark. This kind of new physics was taken into account by Bamert, et. al. [11] during the discrepancy between experiment and SM theory in the R_b ratio. They analyzed a broad class of models in order to explain the discrepancy, and they considered those models in which new $Zb\overline{b}$ couplings arise at tree level through Z or b quark mixing with new particles.

Our presentation is based on the parametrization of the $Zf\overline{f}$ vertex in an independent model formulation. Therefore these results can be used for different quark representations like singlet down quark, vector doublets model, mirror fermions and self conjugated triplets, etc. The parametrization of the vertex in a general way has been reviewed by Barger et. al. [1], [4] as well as Cotti and Zepeda [9]. The LEP precision test parameters that we use are the total Z width Γ_Z, R_l and R_b.

The procedure to get bounds on the mixing angles is the following. First, we analyze the $Zf\overline{f}$ vertex as obtained after a rotation of a general quark

multiplet (common charge) into mass eigenstates. In particular, we write down the neutral current terms for the bottom quarks, which are assumed to be mixed. With these expressions we can evaluate the tree level contribution to the process $Z \to b\bar{b}$; we enclose this new contribution within the coupling constants g_V (vectorial) and g_A (axial). We then write down Γ_Z, R_l and R_b including the new contributions, and we obtain bounds on the new parameters by using the experimental values from LEP and SLC [12]. Finally, we do a χ^2 analysis and find the allowed region in the plane of the new parameters Λ_L and Λ_R introduced. We also use the result obtained by Grossman et. al., involving $B \to \nu\overline{\nu}X_s$[4], in order to narrow down the bounds in the contour plots.

2 Precision test parameters

To restrict new physics, we will use parameters measured at the Z pole. These parameters are the total decay width of the Z boson Γ_Z, the fractions $R_b = \Gamma\left(Z \to b\bar{b}\right)/\Gamma\left(Z \to hadrons\right)$ and $R_l = \Gamma\left(Z \to hadrons\right) / \Gamma\left(Z \to l\bar{l}\right)$ [10], [12]. Considering the new physics (NP) and the SM couplings, we can write

$$\begin{aligned}\Gamma\left(Z \to b\bar{b}\right) &= \frac{G_F m_Z^3}{6\sqrt{2}\pi}\left[\frac{3\beta-\beta^3}{2}\left(g_V^{SM}+g_V^{NP}\right)^2+\beta^3\left(g_A^{SM}+g_A^{NP}\right)^2\right] \\ &\times \; N_C R_{QCD+QED} \end{aligned} \tag{1}$$

where N_C is the number of colors, $R_{QCD+QED}$ are the QCD and QED corrections, and $\beta = \sqrt{1-4\frac{m_b^2}{m_Z^2}}$ is the kinematic factor [10] with $m_b = 4.7$ GeV. We also are taking into account the oblique and vertex contributions to $g_{V,A}^{SM}$ giving by the top quark and Higgs boson. For our purpose, It is convenient to separate the SM and NP contributions as follows:

$$\Gamma\left(Z \to b\bar{b}\right) = \Gamma_b^{SM}\left(1+\delta_b^{NP}\right). \tag{2}$$

The symbol δ_b^{NP} is given by:

$$\delta_b^{NP} = \frac{(3-\beta^2)\left[\left(g_V^{NP}\right)^2+2g_V^{NP}g_V^{SM}\right]+2\beta^2\left[\left(g_A^{NP}\right)^2+2g_A^{NP}g_A^{SM}\right]}{(3-\beta^2)\left(g_V^{SM}\right)^2+2\beta^2\left(g_A^{SM}\right)^2}. \tag{3}$$

This equation could be written using the new physics parameters $\Lambda_{L(R)}$ that were introduced in the eq. (18), through the relationships $g_A^{NP} = \Lambda_L - \Lambda_R$ and $g_V^{NP} = \Lambda_R + \Lambda_L$

Similarly, the Z decay into hadrons after considering the NP, can be written as:

$$\begin{aligned} \Gamma\left(Z \rightarrow hadrons\right) &= 2\Gamma_u^{SM} + 2\Gamma_d^{SM} + \Gamma_b \\ &= \Gamma_{had}^{SM}\left(1 + \frac{\Gamma_b^{SM}}{\Gamma_{had}^{SM}}\delta_b^{NP}\right). \end{aligned} \tag{4}$$

Here, only Γ_b gets NP corrections because only the SM bottom mixes with the exotic quark. Therefore, the Z partial decay into d and s quarks remains unchanged.

On the other hand, Γ_Z is equal to

$$\Gamma_Z = 3\Gamma\left(Z \rightarrow \nu\overline{\nu}\right) + 3\Gamma\left(Z \rightarrow l\overline{l}\right) + \Gamma\left(Z \rightarrow hadrons\right) \tag{5}$$

which again is re-written, with the eq.(4), as follows:

$$\Gamma_Z = \Gamma_Z^{SM}\left(1 + \frac{\Gamma_b^{SM}}{\Gamma_Z^{SM}}\delta_b^{NP}\right). \tag{6}$$

Using the above equations, for R_l and R_b we obtain the following expressions:

$$\begin{aligned} R_l &= R_l^{SM}\left(1 + R_b^{SM}\delta_b^{NP}\right), \\ R_b &= R_b^{SM}\left[1 + \delta_b^{NP}\left(1 - R_b^{SM}\right)\right]. \end{aligned} \tag{7}$$

In a general way, R_b is mainly a measure of $\left|g_L^b\right|^2 + \left|g_R^b\right|^2$; therefore, the fraction R_b is very sensitive to anomalous couplings of the b quark.

3 The model

Following closely the notation of ref. [9], if we have a multiplet $\Psi_{a=L,R}^O$ with n_a ordinary fermions and m_a exotic fermions with the same electric charge q:

$$\Psi_a^O = U_a\Psi_a, \quad \Psi_a = \left(\begin{array}{c} \Psi_l \\ \Psi_h \end{array}\right)_a \tag{8}$$

where U_a is the unitary matrix that rotates the mass eigenstate Ψ_a into the interaction eigenstate Ψ_a^O. $\Psi_{l(h)}$ means ordinary or light (exotic or heavy) fermions. U_a can be further the composed as follows[9]:

$$U_a = \begin{pmatrix} A & E \\ F & G \end{pmatrix}_a \tag{9}$$

where

$$\left(U^+U\right)_a = \begin{pmatrix} A^+A + F^+F & A^+E + F^+G \\ E^+A + G^+F & E^+E + G^+G \end{pmatrix}_a = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad . \tag{10}$$

If we suppose that the up quark sector of the SM is diagonal and that there are no exotic quarks, then A_L corresponds to the classical Kobayashi-Maskawa matrix. In the SM this matrix is unitarity, whereas in our model it is not:

$$\left(A^+A\right)_L = I - \left(F^+F\right)_L . \tag{11}$$

F_L corresponds to the mixing of the ordinary-exotic quarks. As mentioned, A_L is not quite unitary and the factor $(F^+F)_L$ indicates Flavor Changing transitions in the light-light sector.

The neutral current Lagrangian for the multiplet Ψ is given by

$$\begin{aligned} -\mathcal{L}^{NC} &= \frac{e}{c_w s_w} \sum_{a=L,R} \overline{\Psi^O}_a \gamma^\mu D_a \Psi_a^O Z_\mu^0, \\ &= \frac{e}{c_w s_w} \sum_{a=L,R} \overline{\Psi}_a \gamma^\mu U_a^+ D_a U_a \Psi_a Z_\mu^0 \end{aligned} \tag{12}$$

where $s_w = \sin\theta_w$ and D_a are diagonal matrices which contain the couplings of the neutral Z^0 gauge boson to the matter fields; they have the form:

$$\begin{aligned} D_a &= \left(T_3 - Q s_w^2\right)_a , \\ &= \begin{pmatrix} t_{30} - q s_w^2 & 0 \\ 0 & t_{3E} - q s_w^2 \end{pmatrix}_a \end{aligned} \tag{13}$$

where T_{3a} and Q are the matrices of the isospin charges and the electric charge, respectively. t_{30} and t_{3E} are the standard and exotic weak isospin 3rd componend of the multiplets. Using the unitarity relations of the U_a

matrix from the eq. (10), the product $(U^{+}DU)_{a=L,R}$ in eq. (12) can be written as:

$$
\begin{aligned}
\left(U^{+}DU\right)_{L} &= \begin{pmatrix} F^{+}F & -A^{+}E \\ -E^{+}A & -E^{+}E \end{pmatrix}_{L} (t_{3E}-t_{30})_{L} + T_{3L} - Qs_{w}^{2}, \\
\left(U^{+}DU\right)_{R} &= \begin{pmatrix} F^{+}F & F^{+}G \\ G^{+}F & G^{+}G \end{pmatrix}_{R} t_{3ER} - Qs_{w}^{2}
\end{aligned}
\tag{14}
$$

The neutral current Lagrangian in the light-light sector can be written as [9]:

$$
\mathcal{L}^{NC} = -\frac{e}{c_{w}s_{w}} \sum_{a=L,R} \overline{\Psi}_{l,a}\gamma^{\mu}K_{a}\Psi_{l,a}Z_{\mu}^{0} \tag{15}
$$

where

$$
\begin{aligned}
K_{L} &= \left(F^{+}F\right)_{L}(t_{3EL}-t_{30L}) + I_{3\times 3}\left(t_{30L}-qs_{w}^{2}\right), \\
K_{R} &= \left(F^{+}F\right)_{R} t_{3ER} - I_{3\times 3}\, qs_{w}^{2}.
\end{aligned}
\tag{16}
$$

For the SM with three generations, $K_{L,R}$ are 3×3 matrices. They can be produced FC transitions at the tree level depending of $F_{L,R}$ entries which are the mixing angles of the ordinary and exotic fermions.

In this work, we only consider one *bottom* exotic quark (i.e. not mixing with d and s). Then, U_{a} and the $F^{\dagger}F$ product become:

$$
U_{a} = \begin{pmatrix} & & & 0 \\ & A & & 0 \\ & & & -s_{a} \\ 0 & 0 & s_{a} & c_{a} \end{pmatrix} \quad , \quad (F^{\dagger}F)_{a} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sin^{2}\theta_{a} \end{pmatrix}. \tag{17}
$$

where $\sin\theta_{L,R}$ represent the mixing between bottom quark with the exotic ones. Therefore, the coupling bbZ gets modified by the $\Lambda_{L,R}$ factors:

$$
\begin{aligned}
K_{L}^{b} &= \Lambda_{L} + t_{30L} - qs_{w}^{2}, \\
&= \sin^{2}\theta_{L}\left(t_{3EL}+\frac{1}{2}\right) + \left(-\frac{1}{2}+\frac{1}{3}s_{w}^{2}\right), \\
K_{R}^{b} &= \Lambda_{R} - qs_{w}^{2}, \\
&= \sin^{2}\theta_{R}\, t_{3ER} + \left(\frac{1}{3}s_{w}^{2}\right).
\end{aligned}
\tag{18}
$$

4 Results

With the expressions for Γ_Z, R_l and R_b in terms of the new physics contribution in section 3, and with the experimental data from LEP we get bounds on the parameters $\Lambda_{L,R}$ introduced in eq. (18). The experimental data that we used for the LEP parameters, as well as their SM values are in Table 1 [10], [12].

We do a χ^2 fit of the observables Γ_Z, R_l and R_b, and then we proceed to obtain bounds on the parameters $\Lambda_{L,R}$ by taking on values in the best region allowed for them at 95% C.L. This region is displayed in the figure. In order to get a more restrictive region we use the bound $|\Lambda_{L,R}| < 0.0018$ obtained by Grossman et.al. [4], which is represented by straight lines in the figure. The intersection between the two regions is given by:

$$\begin{aligned} -1.094 \times 10^{-4} &\leq \Lambda_L \leq 1.086 \times 10^{-4}, \\ -1.8 \times 10^{-3} &\leq \Lambda_R \leq 1.8 \times 10^{-3}. \end{aligned} \tag{19}$$

We note that for Λ_L the region is more restrictive than the one obtained by Grossman et. al. [4], while the Λ_R parameter is not modified.

If we consider only mixing between an exotic bottom quark with the third SM family, independent of any group representations, it is given by a 2×2 unitary matrix for left- and right-handed fermions. The couplings for several $SU(2)_L$ representations are given in table 2. We can use these bounds in order to get constraints for the left and right mixing angles of each model. They are shown in table 3.

Summarizing, we have used the fractions Γ_Z, R_l and R_b to obtain bounds on the mixing angles of new quark bottom-type representations with the SM bottom quark. Taking into account the results of Grossman et. al.[4], we have gotten the allowed intervals $-1.094 \times 10^{-4} \leq \Lambda_L \leq 1.086 \times 10^{-4}$ and $-1.8 \times 10^{-3} \leq \Lambda_R \leq 1.8 \times 10^{-3}$. Our results reduce the allowed region for the parameter Λ_L while the parameter Λ_R is not modified with respect to the results obtained by Grossman et. al.[4]. We may note that the results have been obtained from the tree level contributions, and we can get bounds on the mass of the new quark using oblique corrections [13].

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	Experimentals	**Standard Model**
Γ_Z	2.4939 ± 0.0024	2.49582
R_l	20.765 ± 0.026	20.7468
R_b	0.21656 ± 0.00074	0.215894

Table 1: SM predictions and experimental values measured at LEP for the Γ_Z, R_l and R_b

(t^3_{EL}, t^3_{ER})	$\mathbf{\Lambda}_L$	$\mathbf{\Lambda}_R$	**Model**
$(0,0)$	$\frac{1}{2}\sin^2\theta_L$	0	Vector singlets
$\left(-\frac{1}{2}, -\frac{1}{2}\right)$	0	$-\frac{1}{2}\sin^2\theta_R$	Vector Doublets
$\left(0, -\frac{1}{2}\right)$	$\frac{1}{2}\sin^2\theta_L$	$-\frac{1}{2}\sin^2\theta_R$	Mirror fermions
$(-1,-1)$	$-\frac{1}{2}\sin^2\theta_L$	$-\sin^2\theta_R$	Self-conjugated triplets

Table 2: The parameters $\Lambda_{L,R}$ for different representations according with the quantum numbers in eq.(15)

Model	$\lvert\sin\theta_L\rvert \leq$	$\lvert\sin\theta_R\rvert \leq$
Vector Singlets	4.661×10^{-2}	0
Vector doublets	0	6×10^{-2}
Mirror fermion	4.661×10^{-2}	6×10^{-2}
Self-conjugated triplets	4.679×10^{-2}	4.24×10^{-2}

Table 3: Bounds on the mixing angles for different representations of the exotic quarks

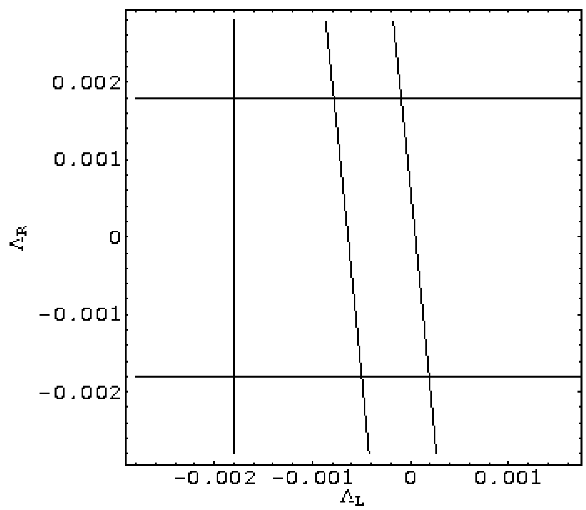

Figure 1: Contour plot represents the allowed region for $\Lambda_L - \Lambda_R$. The straigth lines are the bounds from $B \to \nu\overline{\nu}X$ reported in ref.[4].